Math 70 7.1, 7.2, 7.3 Lesson 21

Rational Expressions

A.K.A. "Algebraic Fractions"

objectives:

1) Find the domain of a rational expression
2) Simplify rational expressions.
3) Multiply rational expressions.
4) Divide rational expressions
5) Review the order of operations
6) Add or subtract rational expressions having like denominators

Math 70 **7.1 Simplifying Rational Expressions**
7.2 Multiplying and Dividing Rational Expressions
7.3 Adding Rational Expressions which have a Common Denominator
Review: Order of Operations

Objectives:

1) Review the order of operations
2) Find the domain of a rational expression
3) Simplify a rational expression (factor and cancel)
4) Multiply rational expressions (factor and cancel)
5) Divide rational expressions (multiply by reciprocal, then factor and cancel)
6) Add rational expressions which have a common denominator (add numerators, keep CD)

The order of operations is a list of rules about the order we do the parts of a calculation containing several parts. Some sources use the acronym PEMDAS.

Step 1: Identify all grouping symbols and resolve them from the inside out. Grouping symbols include purely grouping symbols and grouping symbols which are also operators.

Parentheses ( ), Brackets [ ], and Braces { } – grouping only

Fraction bars – horizontal line creates numerator and denominator groups before divide

For example: $\frac{2-3}{7-4}$ means $(2-3)\div(7-4)$.

Square Roots and other radicals: The radical symbol may enclose a group, before root

For example: $\sqrt{2\cdot3+8}$ means $\sqrt{(2\cdot3+8)}$

Absolute values: The vertical bars may enclose a group, before absolute value

For example: $|3-17\cdot2|$ means $|(3-17\cdot2)|$

Step 2: Exponents, roots, radicals. (Work from left to right.)
Step 3: Multiply and Divide. Work from left to right. (Divide may come before multiply.)
Step 4: Add and Subtract. Work from left to right. (Subtract may come before add.)

Examples

1) Find the domain of $h(x)=\frac{1-4x}{x^3-16x^2+48x}$, then graph the function and use the graph to confirm the domain.

2) Simplify $\dfrac{2x^3+2x^2y-14x^2-14xy}{7-x}$

Perform the indicated operations, then fully simplify if possible.

3) $\dfrac{x^2}{3x^2-5x-2}-\dfrac{2x}{3x+1}\cdot\dfrac{1}{x-2}$

4) $\dfrac{10y^2z-10yz^2}{y^2-z^2}+\dfrac{10yz-15z^2}{2y^2-yz-3z^2}$

5) $\dfrac{8a^3+b^3}{2a^2+3ab+b^2}\div\dfrac{8a^2-4ab+2b^2}{4a^2+4ab+b^2}$

Bonus Challenge problems

6) $\dfrac{9x^3y^5}{(4z)^3}\div\dfrac{(3x^2y)^4}{16xy^6z^4}$

7) $\dfrac{3x^2-5x-2}{3x^4+6x^3+12x^2}\div\dfrac{1}{-18x^3+6x^2-2x}\cdot\dfrac{8-x^3}{27x^3+1}$

8) $\dfrac{4x+36}{5x^3}\div\left(\dfrac{5x^2+39x-54}{3x-12}\cdot\dfrac{x^2-4x}{25x^2-36}\right)$

Math 70

1) Find the domain of $h(x) = \dfrac{1-4x}{x^3 - 16x^2 + 48x}$, then graph the function and use the graph to confirm the domain.

denominator $= 0$

$x^3 - 16x^2 + 48x = 0$

$x(x^2 - 16x + 48) = 0$

$x(x-12)(x-4) = 0$

$\boxed{x \neq 0, 12, 4}$ ← vertical asymptotes

or

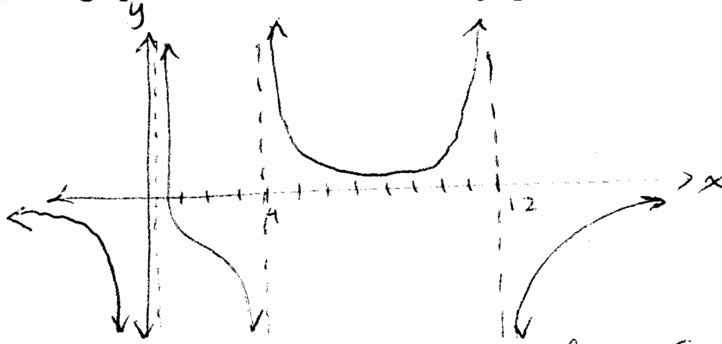
$\boxed{\text{all reals except } 0, 4, 12}$

or

$\boxed{(-\infty, 0) \cup (0,4) \cup (4,12) \cup (12, \infty)}$

+48
$-12 \quad -4$
-16

$-1, -48$
$-2, -24$
$-3, -16$
$-4, -12$



"heartbeat" graph on GC?
change window

XMAX = 15
YMIN = −1
YMAX = 1

2) Simplify $\dfrac{2x^3 + 2x^2y - 14x^2 - 14xy}{7-x}$

$2x^2(x+y) - 14x(x+y)$
$(x+y)(2x^2 - 14x)$
$2x(x+y)(x-7)$

} factor numerator by grouping

$= \dfrac{2x(x+y)(x-7)}{-(x-7)}$ } factor GCF -1 from denominator
$7 - x = -1(-7+x) = -(x-7)$

$= \boxed{-2x(x+y)}$ "cancel" or divide out common factors $\dfrac{x-7}{x-7} = 1$

Math 70

③ $\dfrac{x^2}{3x^2-5x-2} - \dfrac{2x}{3x+1} \cdot \dfrac{1}{x-2}$

subtract second

multiply first

$= \dfrac{x^2}{3x^2-5x-2} - \dfrac{2x}{(3x+1)(x-2)}$

no factors cancel! Leave factored.

factor $3x^2-5x-2$ (or multiply $(3x+1)(x-2)$)

-6, -6, $+1$, -5

$3x^2-6x+x-2$

$3x(x-2)+1(x-2)$

$(x-2)(3x+1)$

It is a common denominator!

$= \dfrac{x^2}{(3x+1)(x-2)} - \dfrac{2x}{(3x+1)(x-2)}$

subtract numerators & keep denominator

$= \dfrac{x^2-2x}{(3x+1)(x-2)}$

simplify by factor and cancel

$= \dfrac{x(x-2)}{(3x+1)(x-2)}$ GCF x

$= \boxed{\dfrac{x}{3x+1}}$

M70

④ $$\frac{10y^2z - 10yz^2}{y^2 - z^2} + \frac{10yz - 15z^2}{2y^2 - yz - 3z^2}$$

not a common denominator!
Try to simplify each fraction first. (factor + cancel)

factor $10y^2z - 10yz^2$

$= 10yz\,(y - z)$ GCF

factor $y^2 - z^2$

$= (y-z)(y+z)$ difference of squares

factor $10yz - 15z^2$

$= 5z\,(2y - 3z)$

factor $2y^2 - yz - 3z^2$

-6

$-3 \quad 2$

-1

rewrite middle term as two like terms

$$\underbrace{2y^2 - 3yz}\; + \underbrace{2yz - 3z^2}$$

$= y(2y - 3z) + z(2y - 3z)$ factor by grouping

$= (2y - 3z)(y + z)$

Rewrite original question in factored form

$$\frac{10yz\,\cancel{(y-z)}}{\cancel{(y-z)}(y+z)} + \frac{5z\,\cancel{(2y-3z)}}{\cancel{(2y-3z)}(y+z)}$$ cancel common factors

$$= \frac{10yz}{y+z} + \frac{5z}{y+z}$$ common denominator!

$$= \frac{10yz + 5z}{y+z}$$ add numerators

$$= \boxed{\frac{5z(2y+1)}{y+z}}$$ factor final answer (hope to cancel)

Math 70

(5) $$\frac{8a^3+b^3}{2a^2+3ab+b^2} \div \frac{8a^2-4ab+2b^2}{4a^2+4ab+b^2}$$

$\uparrow$ divide means multiply by reciprocal

$$= \frac{8a^3+b^3}{2a^2+3ab+b^2} \cdot \frac{4a^2+4ab+b^2}{8a^2-4ab+2b^2}$$

multiply by factor and cancel

factor $8a^3+b^3$

$$= (2a+b)(4a^2-2ab+b^2)$$

sum of cubes

factor $2a^2+3ab+b^2$

$\begin{array}{ccc} & 2 & \\ 2 & \times & 1 \\ & 3 & \end{array}$

$$= \underbrace{2a^2+2ab}+\underbrace{ab+b^2}$$

rewrite middle term using like terms

$$= 2a(a+b)+b(a+b)$$

$$= (a+b)(2a+b)$$

factor $4a^2+4ab+b^2$

$\begin{array}{ccc} & 4 & \\ 2 & \times & 2 \\ & 4 & \end{array}$

$$= \underbrace{4a^2+2ab}+\underbrace{2ab+b^2}$$

$$= 2a(2a+b)+b(2a+b)$$

$$= (2a+b)(2a+b) \text{ or } (2a+b)^2$$

factor $8a^2-4ab+2b^2$

$\begin{array}{ccc} & 16 & \\ & \times & \\ & -4 & \end{array}$

no! $\Rightarrow$ GCF 2!

$(4a^2-2ab+b^2)$ is a prime trinomial.

$$= 2(4a^2-2ab+b^2)$$

$\begin{array}{ccc} & 4 & \\ & \times & \\ & -2 & \end{array}$ no!

Rewrite original question:

$$= \frac{(2a+b)\cancel{(4a^2-2ab+b^2)}}{(a+b)\cancel{(2a+b)}} \cdot \frac{\cancel{(2a+b)}(2a+b)}{2\cancel{(4a^2-2ab+b^2)}}$$

cancel common factors

$$= \boxed{\frac{(2a+b)(2a+b)}{2(a+b)}}$$

or

$$= \boxed{\frac{(2a+b)^2}{2(a+b)}}$$

M70

(6) Perform the indicated operations and fully simplify

$$\frac{9x^3y^5}{(4z)^3} \div \frac{(3x^2y)^4}{16xy^6z^4}$$

order of operations: parentheses & grouping symbols

$$= \frac{9x^3y^5}{4^3z^3} \div \frac{3^4(x^2)^4y^4}{16xy^6z^4}$$ exponent law $(ab)^n = a^nb^n$

$$= \frac{9x^3y^5}{64z^3} \div \frac{81x^8y^4}{16xy^6z^4}$$ exponent law $(a^n)^m = a^{n\cdot m}$

order of operations: exponents

$$= \frac{9x^3y^5}{64z^3} \div \frac{81x^7}{16y^2z^4}$$ exponent law $\frac{x^n}{x^m} = x^{n-m} = \frac{1}{x^{m-n}}$

order of operations: multiply & divide from left to right

$$= \frac{9x^3y^5}{64z^3} \cdot \frac{16y^2z^4}{81x^7}$$ multiply by reciprocal

$$= \frac{9}{81} \cdot \frac{16}{64} \cdot \frac{x^3}{x^7} \cdot y^5 \cdot y^2 \cdot \frac{z^4}{z}$$ exponent law $x^n \cdot x^m = x^{n+m}$

$$= \frac{1}{9} \cdot \frac{1}{4} \cdot \frac{1}{x^4} \cdot y^7 \cdot z^3$$

$$= \boxed{\frac{y^7z^3}{36x^4}}$$

M70

(7) Perform the indicated operations and fully simplify.

$$\frac{3x^2-5x-2}{3x^4+6x^3+12x^2} \div \frac{1}{-18x^3+6x^2-2x} \cdot \frac{8-x^3}{27x^3+1}$$

order of operations ; multiply & divide from left to right

$$= \frac{3x^2-5x-2}{3x^4+6x^3+12x^2} \cdot \frac{-18x^3+6x^2-2x}{1} \cdot \frac{8-x^3}{27x^3+1}$$

factor everything completely:

A → $3x^2-5x-2$ 3 terms = trinomial, leading coefficient $3 \neq 1$ $3(-2)$ -6 ; -6 $+1$; -5

$= \underbrace{3x^2-6x}+\underbrace{x-2}$ rewrite middle term using like terms

$= 3x(x-2)+1(x-2)$ factor by grouping

$=(x-2)(3x+1)$

B → $3x^4+6x^3+12x^2$ GCF $3x^2$

$= 3x^2(x^2+2x+4)$ 4 ; 2 trinomial is prime

C → $-18x^3+6x^2-2x$ GCF $-2x$

$= -2x(9x^2-3x+1)$ 9 ; -3 trinomial is prime

D → $8-x^3$

$= -x^3+8$ Write in standard form

$= -(x^3-8)$ GCF (-1)

$= -(x-2)(x^2+2x+4)$ Difference of cubes $a^3-b^3=(a-b)(a^2+ab+b^2)$
S O AP

E ⇒ $27x^3+1$

$= (3x+1)(9x^2-3x+1)$ Sum of cubes $a^3+b^3=(a+b)(a^2-ab+b^2)$.

M70

(7) continued

Rewrite in factored form:

$$= \frac{(x-2)(3x+1)}{3x^2(x^2+2x+4)} \cdot \frac{(-2x)(9x^2-3x+1)}{1} \cdot \frac{(-1)(x-2)(x^2+2x+4)}{(3x+1)(9x^2-3x+1)}$$

cancel common factors: $\frac{fac}{fac} = 1$

$$= \frac{(x-2)\cancel{(3x+1)}(-2x)\cancel{(9x^2-3x+1)}(-1)(x-2)\cancel{(x^2+2x+4)}}{3x^2\cancel{(x^2+2x+4)}\cancel{(3x+1)}\cancel{(9x^2-3x+1)}}$$

$$= \frac{2x(x-2)^2}{3x^2}$$

$$= \boxed{\frac{2(x-2)^2}{3x}}$$

M

(8) Perform the indicated operations and fully simplify.

$$\frac{4x+36}{5x^3} \div \left(\frac{5x^2+39x-54}{3x-12} \cdot \frac{x^2-4x}{25x^2-36} \right)$$

Order of operations: parentheses

Factor everything

A → $4x+36$ GCF 4

$= 4(x+9)$

B → $5x^2+39x-54$ trinomial leading coefficient $\neq 1$

Guess + check

$(5x-6)(x+9)$

1×54
2×27
3×18
6×9

Double X

$5(-54)$
-270
-6 45
39

-1×270
-2×135
-3×90
-6×45

$5x^2-6x+45x-54$

$= x(5x-6)+9(5x-6)$

$= (5x-6)(x+9)$

C → $3x-12$ GCF 3

$= 3(x-4)$

D → x^2-4x GCF x

$= x(x-4)$

E → $25x^2-36$ difference of squares

$= (5x-6)(5x+36)$

Rewrite: and cancel

$$= \frac{4(x+9)}{5x^3} \div \left(\frac{\cancel{(5x-6)}(x+9)}{3\cancel{(x-4)}} \cdot \frac{x\cancel{(x-4)}}{\cancel{(5x-6)}(5x+6)} \right)$$

$$= \frac{4(x+9)}{5x^3} \div \frac{x(x+9)}{3(5x+6)}$$

multiply by reciprocal + cancel

$$= \frac{4\cancel{(x+9)}}{5x^3} \cdot \frac{3(5x+6)}{x\cancel{(x+9)}}$$

$$= \frac{4 \cdot 3(5x+6)}{5x^3 \cdot x} = \boxed{\frac{12(5x+6)}{5x^4}}$$